Time reversal space time block coding with channel estimation errors

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Abstract

Time reversal (TR) space time block coding (STBC) is a novel time domain technique to mitigate inter-symbol interference (ISI) in frequency-selective fading channels. This paper derives time-domain TR equations without requiring the q-operator, and presents simulation results for the case when channel estimates are not accurately determined through training. The time-domain TR equations have particular relevance towards the architecture of a TR-STBC system implementation whilst the calculated effects of channel model inaccuracy impact system packet structure and protocol issues.

1. Introduction

The Alamouti scheme [1] is a widely-investigated transmit diversity technique to combat flat fading channels. This has been extended in [2-4] into a time domain block symbol processing scheme called time-reversal (TR) space-time block coding (STBC).

TR-STBC lends itself to decoupled and parallel equalisation schemes for frequency selective fading channels. It has been designed to simplify the receive decoding process through coding and ordering of transmit sequences.

This paper reformulates the TR-STBC equations into implementation relevant time-domain structures and investigates TR-STBC through simulation of the effect of training length (and therefore channel model accuracy) on system performance.

2. System Model

Let us consider the case when there are two transmit antennas at the transmitter and one receive antenna at the receiver. Assume that in the first burst the block of symbols transmitted by antenna-1 is

$$S_1 = \{d_1(0), d_1(1), \ldots\ldots\ldots d_1(N)\} \quad \ldots\ldots\ldots \quad (1)$$

and that the block of symbols transmitted simultaneously by antenna-2 is

$$S_2 = \{d_2(0), d_2(1), \ldots\ldots\ldots d_2(N)\} \quad \ldots\ldots\ldots \quad (2)$$

As can be seen each antenna transmits N+1 symbols in the first burst. In the second transmission burst, antenna-1 transmits a time reversed, complex conjugated and sign inverted version of the block transmitted by antenna-2 in the first burst. Similarly, in the second burst antenna-2 transmits a time reversed and complex conjugated version of the block transmitted by antenna-1 in the first burst. Mathematically, the time reversal and complex conjugation operation can be represented as

$$\widetilde{S_1} = \{d_1^*(N), d_1^*(N-1), \ldots\ldots, d_l^*(0)\} = \{D_1(0), D_1(1), \ldots. D_1(N)\} \quad \ldots\ldots\ldots\ldots\ldots\ldots \quad (3)$$

$$-\widetilde{S_2} = \{d_2^*(N), -d_2^*(N-1), \ldots\ldots, -d_2^*(0)\} = \{D_2(0), D_2(1), \ldots\ldots, D_2(N)\} \quad \ldots\ldots\ldots\ldots\ldots \quad (4)$$

where $\widetilde{S_1}$ is the time reversed and complex conjugated version of S_1 and similarly for $\widetilde{S_2}$. The negation prefixed before $\widetilde{S_2}$ indicates that what is transmitted is actually $-\widetilde{S_2}$ rather than $\widetilde{S_2}$. The table below summarises the transmitted blocks from the two antennas during two sequential transmission bursts.

	Burst 1	Burst 2
Antenna 1	S_1	$-\widetilde{S_2}$
Antenna 2	S_2	$\widetilde{S_1}$

Let the channel impulse response from antenna-1 to the receive antenna be fixed at 4-taps for the sake of the following analysis. This is denoted by tap weights g_0, g_1, g_2 and g_3. A 4-tap response is chosen arbitrarily to illustrate the principle whereas in practice the response may be either longer or shorter. Similarly, let the channel impulse response from antenna-2 to the receive antenna be denoted by tap weights p_0, p_1, p_2 and p_3. We assume that the channel is stationary during a block of symbols and throughout the two bursts. The signal received at the single receive antenna for the burst 1 can now be expressed as:

$$r_1(t) = g_0 d_1(t) + g_1 d_1(t-1) + g_2 d_1(t-2) + g_3 d_1(t-3) + p_0 d_2(t) + p_1 d_2(t-1) + p_2 d_2(t-2) + p_3 d_2(t-3) + n_1(t)$$
$$for \quad t = 0 \ to \ N \quad \ldots\ldots\ldots\ldots \quad (5)$$

where $n_1(t)$ is assumed to be white noise with zero mean and variance σ_0^2. Similarly, the received signal for the second burst can be expressed as:

$$r_2(t) = g_0 D_2(t) + g_1 D_2(t-1) + g_2 D_2(t-2) + g_3 D_2(t-3) + p_0 D_1(t) + p_1 D_1(t-1) + p_2 D_1(t-2) + p_3 D_1(t-3) + n_2(t)$$
$$for \quad t = 0 \ to \ N \quad \ldots\ldots\ldots\ldots \quad (6)$$

The second burst of received signal is then time reversed and complex conjugated as follows:

$$r_3(t) = r_2^*(N-t) \quad for \quad t = 0 \quad to \quad N \quad \ldots. \quad (7)$$

Thus, the time reversed and complex conjugated signal can be expressed as:

$$r_3(t) = r_2^*(N-t) = g_0^* D_2^*(N-t) + g_1^* D_2^*(N-t-1) + g_2^* D_2^*(N-t-2) + g_3^* D_2^*(N-t-3) + p_0^* D_1^*(N-t) + p_1^* D_1^*(N-t-1) + p_2^* D_1^*(N-t-2) + p_3^* D_1^*(N-t-3) + n_2^*(N-t)$$
$$for \quad t = 0 \ to \ N \quad \ldots\ldots\ldots\ldots \quad (8)$$

As

$$D_2(t) = -d_2^*(N-t)$$
$$D_1(t) = d_1^*(N-t) \quad \ldots\ldots\ldots\ldots \quad (9)$$

we can see that

$$D_2^*(N-t-k) = -d_2(N-(N-t-k)) = -d_2(t+k)$$
$$D_1^*(N-t-k) = d_1(N-(N-t-k)) = d_1(t+k)$$
$$for \ any \ arbitrary \ integer \ k. \quad \ldots\ldots \quad (10)$$

Thus, the received time reversed and complex conjugated measurement can be written as

$$r_3(t) = r_2^*(N-t) = -g_0^* d_2(t) g_1^* d_2(t+1) - g_2^* d_2(t+2) - g_3^* d_2(t+3) + p_0^* d_1(t) + p_1^* d_1(t+1) + p_2^* d_1(t+2) + p_3^* d_1(t+3) + n_2^*(N-t)$$
$$for \quad t = 0 \ to \ N \quad \ldots\ldots\ldots\ldots \quad (11)$$

3. Linear Combining

To illustrate linear combining in the time domain we can take a slight shortcut but without compromising the elegance of the proof. We assume that the channel impulse response is only 2 taps in length so that the number of terms to be handled is reduced. When the number of taps is increased, this will only increase the extra terms of a similar nature, whilst the concept will not change. The original work in [2-4] uses the q-operator which avoids carrying over too many terms. The idea in this section is to formulate all the equations in discrete time in order to aid implementation and understanding. However, if we wished to compare the received sample expressions in the q-domain we can simply re-write the expressions for r1(t) and r3(t) as

$$\begin{bmatrix} r_1(t) \\ r_3(t) \end{bmatrix} = \begin{bmatrix} g(q^{-1}) & p(q^{-1}) \\ p^*(q) & -g^*(q) \end{bmatrix} \begin{bmatrix} d_1(t) \\ d_2(t) \end{bmatrix} + \begin{bmatrix} n_1(t) \\ n_3(t) \end{bmatrix} \quad \ldots\ldots \quad (12)$$

where

$$g(q^{-1}) = g_0 + g_1 q^{-1} + g_2 q^{-2} + g_3 q^{-3}$$
$$p(q^{-1}) = p_0 + p_1 q^{-1} + p_2 q^{-2} + p_3 q^{-3}$$
$$g^*(q) = g_0^* + g_1^* q + g_2^* q^2 + g_3^* q^3$$
$$p^*(q) = p_0^* + p_1^* q + p_2^* q^2 + p_3^* q^3 \quad \ldots\ldots \quad (13)$$

Again following the simple approach of assuming that there are only 2 taps in the channel impulse response, i.e.

$$g(q^{-1}) = g_0 + g_1 q^{-1}$$
$$p(q^{-1}) = p_0 + p_1 q^{-1}$$
$$g^*(q) = g_0^* + g_1^* q$$
$$p^*(q) = p_0^* + p_1^* q \quad \ldots\ldots\ldots\ldots \quad (14)$$

To perform linear combination, the received data samples are filtered in the following manner:

To generate Antenna 1 measurements $z_1(t)$, pass $r_1(t)$ through a non-causal filter with coefficients g_0^* and g_1^*, pass $r_3(t)$ through a causal filter with coefficients p_0 and p_1, and sum the results.

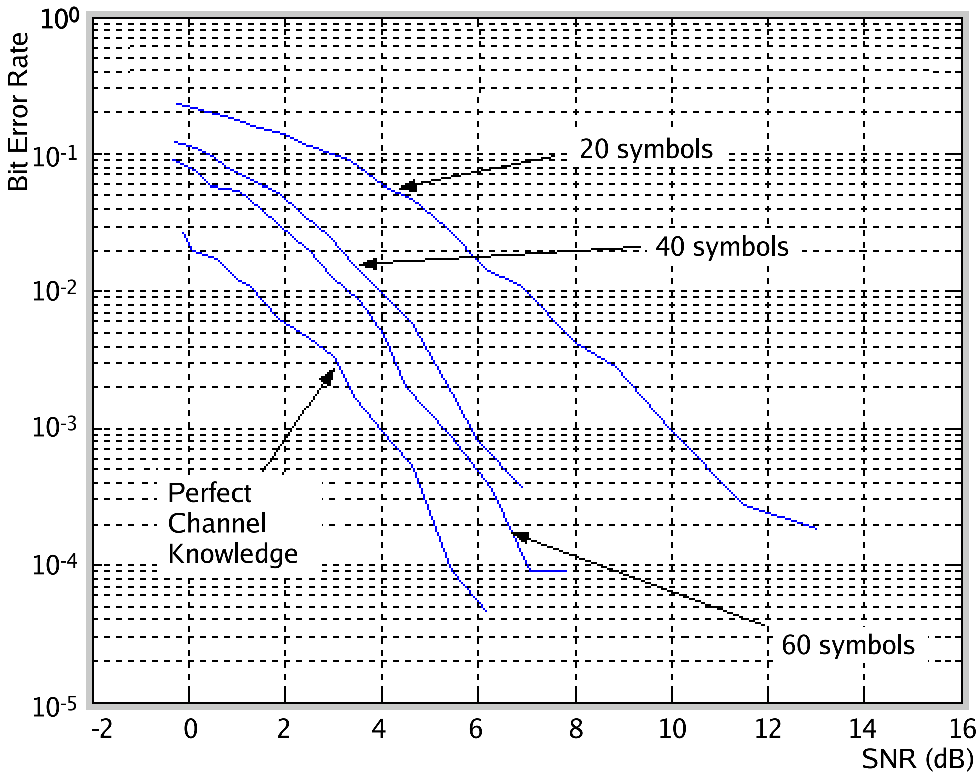

Fig. 1: Bit error rate plotted against signal-to-noise ratio for training sequence lengths of 20, 40 and 60 symbols and for perfect ("non-estimated") training used for channel estimation in a TR-STBC system.

To generate Antenna 2 measurements $z_2(t)$, pass $r_1(t)$ through a non-causal filter with coefficients p_0^* and p_1^*, pass $r_3(t)$ through a causal filter with coefficients $-g_0$ and $-g_1$, and sum up the results.

$$z_1(t) = g_0^* r_1(t) + g_1^* r_1(t+1) + p_0 r_3(t) + p_1 r_3(t-1) \quad \text{..............................} \quad (15)$$
$$z_2(t) = p_0^* r_1(t) + p_1^* r_1(t+1) - g_0 r_3(t) - g_1 r_3(t-1) \quad \text{................................} (16)$$

Substituting the expressions for $r_1(t)$ and $r_3(t)$ and performing algebraic simplifications results in the following expressions:

$$z_1(t) = \gamma_0 d_1(t) + \gamma_1 d_1(t-1) + \gamma_1^* d_1(t+1) \quad \ldots \ (17)$$
$$z_2(t) = \gamma_0 d_2(t) + \gamma_1 d_2(t-1) + \gamma_1^* d_2(t+1) \quad \ldots \ (18)$$

As formulated in [2-4] for a general n_r-tap channel, the complex conjugate coefficients are given by

$$\gamma(q, q^{-1}) = \gamma_{n_r}^* q^{n_r} + \gamma_{n_r-1}^* q^{n_r-1} + \ldots\ldots + \gamma_0 + \ldots\ldots + \gamma_{n_r-1} q^{-(n_r-1)} + \gamma_{n_r} q^{-n_r}$$
$$\equiv h_1^*(q) h_1(q^{-1}) + h_2^*(q) h_2(q^{-1}) \quad \ldots \ (19)$$

It is easy to see that for a 4 tap channel response we have

$$n_r = 3$$
$$\gamma_0 = p_0^2 + p_1^2 + p_2^2 + p_3^2 + g_0^2 + g_1^2 + g_2^2 + g_3^2$$
$$\gamma_1 = p_0^* p_1 + p_1^* p_2 + p_2^* p_3 + g_0^* g_1 + g_1^* g_2 + g_2^* g_3$$
$$\gamma_2 = p_0^* p_2 + p_1^* p_3 + g_0^* g_2 + g_1^* g_3$$
$$\gamma_3 = p_0^* p_3 + g_0^* g_3 \quad \text{...} \quad (20)$$

4. Simulation Results

The simulation scenario consists of two transmit antennas and one receive antenna. A 4-tap channel model is assumed for the simulation and each symbol is oversampled 5 times. 55 bits (BPSK modulation) are transmitted in the forward and reverse data time slots from each antenna. There is a window of 5 bits between the forward and reverse data blocks when known data (–1s) is transmitted.

The time-domain equations in section 3 are applied to the data in a MATLAB simulation, with a view to determining the effect of channel estimation accuracy. In this case channel estimation accuracy is varied through adjusting the length of training sequence used to determine the channel.

Fig. 1 illustrates the effects of channel estimation accuracy on the achieveable bit error rate (BER) versus signal to noise ratio (SNR). As can be observed the best performance is obtained when the number of training bits used is increased. The plots are shown for 20, 40 and 60 training bits, and for perfect channel knowledge.

The graph shows that moving from 20 to 40 training bits yields a relatively large improvement in system performance whereas increasing still further to 60 bits yields proportionately less improvement.

5. Conclusion

Previous research has introduced the TR concept to the space time processing research field. The method has proven particularly suitable for the mitigation of inter-symbol interference.

This paper reformulates the published TR equations into a time-domain representation suitable for implementation. The implementation equations have then been simulated to account for different degrees of implementation accuracy. Results indicate the degree of overall numerical accuracy required to maintain a given level of ISI immunity for given simulation channel.

6. References

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